

# COMMENTS

## to the presentation

### “A Financial Instruments Pricing Model as the Physical Two Players Problem”

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#### Slide 2 – Fundamentals of the classical finance theory

The first attempt to develop a mathematical model for describing the process of statistical market pricing dates back to 1900. In that year Louis Bachelier published his PhD thesis entitled “The theory of speculation”. One of the propositions of the work stated the independence of today’s price changes of yesterday’s. Another proposition assumed that price changes are distributed in accordance with the normal (Gaussian) law.

In the second half of the XX century the Bachelier work was referred to due to the development of a few financial models which subsequently became the pillars of the classical finance theory: the modern portfolio theory (Harry Markowitz, 1952), the capital asset pricing model (William Sharpe, 1964) and the option pricing model (Fischer Black and Myron Scholes, 1973).

It is needless to provide detailed descriptions of those models in this presentation. But what is of crucial importance is that their common suppositions are consistent with those stated in 1900 by Bachelier: price changes are *independent* and *normally distributed*, i.e. described by the classical white noise model leading to the random walk pricing process.

#### Slide 3 – Benoit Mandelbrot’s studies

In the early 1960s Benoit Mandelbrot demonstrated that price changes are actually *not* normally distributed [5, 30-32, 42]. He also opined that they may be *not* independent. In his papers Mandelbrot introduced two parameters:  $H$ , varying in the range from 0 to 1 and being responsible for the presence of “market memory”, or, in other words, for the dependence of today’s price changes of yesterday’s; and  $\alpha$ , varying from 0 to 2 and defining the degree of non-Gaussianity. According to Mandelbrot, it is only the case of  $H = 0.5$  and

$\alpha = 2$  that corresponds to the Bachelier model, which is rarely observed in the real markets.

It should also be noted that the non-Gaussianity and market memory were shown by Mandelbrot to take place at different time scales, be it minutely, daily or weekly data. That phenomenon was named *scaling* of price changes.

As a result, Mandelbrot came to the conclusion that price changes are described by the so-called *fat-tailed distributions* [9, 31]. The normal (Gaussian) probability distribution is merely a particular case ( $\alpha = 2$ ) of one of the subclasses of the fat-tailed distributions, the so-called stable distributions, also known as Levy-stable, or  $\alpha$ -stable [35].

In the figures of this slide there are examples of the probability distributions of price (rate) changes of several financial instruments at different time scales, including the tick scale (for all the data, except the currency pair, we processed differences of the logarithms of the data – just to take into account the inflation). As it is seen, in all the cases there is a difference between the real probability distribution and the normal approximation to it – the large changes happen more frequently than it is predicted by the classical financial models, which is reflected in the presence of the fat tails in the probability distributions. According to the estimates based on methods [24, 26], the value of  $\alpha$  varies, roughly, between 1 and 2.

Due to the findings of Mandelbrot, among the professional community it is now a common place that price changes are not normally distributed, and market professionals realise that real price changes are much more unpredictable than they are expected to be according to the classical models [7, 15, 16, 20]. However, the issue of market memory, or the dependence of successive price changes, is still under discussion [31].

#### **Slides 4-5 – State of the art (econophysics)**

So far the most successful results in developing adequate market pricing models have been obtained by specialists working in *econophysics* – an interdisciplinary branch of knowledge that intends to use the methodology of (statistical) physics for the analysis of economical data [33, 39, 40]. However, nowadays, in our opinion, the methodology applied in the mainstream of econophysics is *not physical*. Instead, those models are more appropriate to describe as “*economathematical*”, as they are developed with involving

purely mathematical techniques which leave the physical essence of the pricing phenomenon unexplained. Typically one takes a predefined deterministic model engaged from some or other area of physics, adapts it some or another way and supplies with a set of stochastic parameters. Examples of such approaches are the autoregressive heteroscedasticity (a large number of GARCH-based models) [1, 25, 33], truncated Levy flights [33, 34], various adaptations of the quantum mechanics' models [4, 6, 13, 18, 23], percolation [43] and neural network models [28, 46, 47], and others [2, 8, 14, 21, 29, 36, 37, 38, 41, 45, 48]. At the practical stage the modelling is reduced to the *imitation* of the pricing process: it is aimed to tune the parameters of a voluntarily taken model in such a manner that the result of the subsequent simulation would statistically match the real historical market data as much as possible [12, 22]. The point is that because the model has been taken a priori rather than derived from some first principles, it is not a surprise that happen a tiny perturbation in the input data – a recalibration of the model will be required. As a result, those models do not adequately simulate the asset's pricing during all the time, and hence do not exhibit permanent forecast strength.

As a matter of fact, the fitting of real market data into an a priori taken model is an attempt to solve an inverse *meta*-problem. It is well-known that the correct solution of an ordinary (i.e. not meta-) inverse problem, whose solution is a system of unknown functions presented in an equation of motion known in advance, is a complex issue due to the ill-posedness of inverse problems: a tiny perturbation of the input data may radically change the solution. As for the market pricing problem, the situation is incomparably more difficult: the discovery of the genuine physical mechanism of pricing dynamics has nothing to do with the search for a system of unknown functions; instead, it relates to the solution of the meta-problem – the derivation of the unknown equation of motion. Apparently that having no clear insight into the physical meaning of the equation to be sought, it is practically impossible to guess the solution by performing complete enumeration over a priori given models.

In the light of the said above, we conclude that despite the numerous attempts so far there is no physically appropriate statistical model which would provide an adequate description of the financial assets pricing dynamics [32, 33].

## Slides 6-7 – Our approach

In our approach, instead of fitting the market reality into a predetermined model we formulate original first principles and derive a model which is hopefully consistent with the reality. In other words, we solve the direct rather than the inverse meta-problem.

We consider the entire set of market agents as an open stochastic mechanical system of two players, the “bull” and the “bear” (according to the historically established notation), each of whom is “tugging” the price to his side. New information thrown into the market is likened to the energy introduced into the system. The energy alters the players’ alignment of forces, thereby changing the price. The concrete factors that lead to some or another alignment of forces are of no importance: those might be bad weather conditions in certain regions of the planet, changes of the interest rates due to the central banks’ decisions, political instability, etc. All those factors are taken into account in the integral way, thus leading to a macroscopic model [17, 19]. Upon this we assume that the price formation is being carried out simultaneously at several time scales. Because every possible price change is related to an amount of energy that can be introduced into the system, we have to find out in how many times the probability of a price change is greater or less than the probability of the other. This is done via the comparison of the amounts of energy: the more energy is required for a price change, the less likely that the price change will happen. Enumerating over all possible values of the future price change, we determine the least “energy-expensive”, or, in other words, the most probable one.

So, our model is an original stochastic (or probability) equation of motion: for each pair of two successive price changes it yields the probability of transition from the first state – the past – to the second one – the future. Therefore, the solution to the model is a probability distribution, or a probability density function, of the future price change subject to the known previous one. (In whole, the general solution to the model is a family of distributions rather than a single density function, but for short we shall be speaking as if there is only one distribution, assuming the previous price change to be known and fixed.) As the price is being formed simultaneously at several time scales (analogous to minutes, days, months, etc.), there is a separate equation of motion at every scale. Gathering all over the scales is what we call the *scales complexification*.

As we do not demonstrate the explicit form of the equations of motion, we realise that this qualitative description is not the conventional academic way to present results of a study. Furthermore, the given description may seem obscure to make a proper judgement on the approach. However, some researchers may find the abovementioned propositions to be worthy of note for carrying out their own studies. In turn, below we analyse the developed model's properties, which are remarkable due to consistency with the properties of real market data.

### **Slide 8 – Our results**

1. The probability distribution (shown in the figure as a smoothed function that depicts the general form but conceals details to prevent reverse engineering) belongs to a class of asymmetric distributions; upon this, the parameter  $\alpha$  is obviously less than two, i.e. the tails are *fat* (non-Gaussian) [31].

It is also worth to note that the wave-like structure of the solution explains the effect of discontinuity of price changes often observed in real markets. In terms of a classical empirical technique – the technical analysis – these are levels of support and resistance that attract the price, so for the price it is more likely to move and get “nailed” to some or other price level rather than to stop somewhere between the picks.

### **Slide 9 – Our results (continuation)**

2. The asymmetry of the probability distribution allows forecasting not only the absolute value of the future price change (e.g., volatility), but also the sign of the change, i.e. the direction.

### **Slide 10 – Our results (continuation)**

In spite of the asymmetry of the distribution in respect of the zero price change, in general the probability of a positive price change is close to that of a negative one when the absolute value of the price change grows. In other words, there is an asymptotic (“tailed”) symmetry of the distribution, and hence the correlation coefficient between two price changes spaced at a certain lag decays and almost instantly becomes close to zero as the time lag grows (the left figure). Nevertheless, the corresponding correlation field does not resemble to be produced by a white noise, as it evinces a nontrivial

internal structure (the right figure). These results are consistent with the empirical observations on the fast decay of the autocorrelation function when analysing real market data [11, 33, 44], which allows to conclude that

3. There exists a nonlinear dependence (*market memory*) between successive price changes [31].

#### **Slide 11 – Our results (continuation)**

4. The distributions' structure is kept unchanged when varying the scale (in this example the scale in the right figure is two times greater than that in the left one), which is consistent with the property of *scaling* of quote changes of real market instruments discovered by Mandelbrot [31].

#### **Slide 12 – Our results (continuation)**

To check the model for the forecast strength, we have performed tests on six real financial instruments as well as on purely random data generated by the Gaussian white noise. To compare the results, we have also performed tests with randomly permuted original data which thereby had assuredly had no dependence at all.

As the real data we used tick quotes of the currency pair euro-dollar from November 2009 to March 2012 (the data source was the Forex bank "Dukascopy", [www.dukascopy.com](http://www.dukascopy.com)), the oil (Brent), the gold and the wheat from 2011 to 2013, as well as the shares of Google and Apple from September 2011 to December 2013 (the data source was the investment holding company "Finam", [www.finam.ru](http://www.finam.ru)). The mean sample lengths were 351 883, 251 506, 449 731, 67 704, 22 321 and 124 692 successive price changes respectively.

For each nonzero price change  $\Delta x_{\text{prev}}$  we computed the probability distribution of the next price change  $\Delta x_{\text{next}}$ . Having that, we made a probability forecast of the direction of that change and compared it with the real price change's direction that took place on the market. If matched, the number of right forecasts was increased by one; in the opposite case we increased the number of wrong forecasts; in the borderline case, when the real price change was nil, both the numbers of right and wrong forecasts were increased by 0.5. For each month we computed the relative frequency to make

the right guess on the direction of the future price change, and afterwards we calculated the mean relative frequency over months.

Analysis of the results related to the white noise price changes confirms that the relative frequency to forecast the direction is about 50% (or, in other words, the relative frequency's skewness  $Fr - 50\%$  is about nil) both on the original and randomly permuted data.

Unlike that, for the real financial instruments the situation is completely different. While the relative frequency's skewness is still about zero when processing the permuted real data (which is comparable with the results related to the purely random data), there is a statistically significant skewness, at the average more than 100 times greater than that for the random data, when processing the original real price changes.

### **Slide 13 – Our results (end)**

Similar experiments have been carried out on closing prices of minutely (M1, M5, M10, M15, M30), hourly (H1) and daily (D1) quotes. The mean sample lengths were from 176 655 (on M1) to 3 829 (on H1) and 268 (on D1) prices moves. To get stable results on short samples, we used stochastic modelling, performing multiple forecasts (up to 100 repetitions on minutes and 100 000 on hours and days) for each  $\Delta x_{\text{prev}}$ .

As it follows from the figures, when processing the randomly permuted price changes the relative frequency to predict the direction is expectedly oscillating around 50%. Specifically, on all the minute time frames there are fairly zero skewnesses with slightly growing dispersions observed when passing to major scales (particularly, for the shares of Google and Apple). These dispersions are explained by the small numbers of process realisations amplified by the rather short samples on the time frames H1 and D1 compared to the minute frames. Nonetheless, in the case of the original data statistically significant skewnesses, at the average above 1%, are observed through all the time frames and on all the instruments.

Therefore, when forecasting the direction of future price changes, the model detects a dependence of the future change subject to the previous one on all the time frames without being calibrated on historical data. As for the low accuracy of the forecast (the maximum skewness is a bit greater than 4%), as well as the presence of the negative skewnesses for several financial

instruments, we explain these by the lack of calibration of the model. However, the goal of these experiments has been achieved indeed: on diverse time frames we have proved that the model is able to detect dependence in successive price changes of real financial instruments, having no a priori information about the data.

#### **Slide 14 – Conclusion**

Analysis of the properties of the developed pricing model and comparison with the models by other authors permit to conclude that the obtained results are advanced and promising:

1. The model's solution – the *fat-tailed* probability distributions possessing the property of *scaling* and detecting a nonlinear dependence (*market memory*) in successive quote changes – is consistent with the results of the studies of real market data which have been carried out so far [7, 15, 30-33, 45].
2. What makes the model qualitatively different and places it beyond the others are its physical foundations: the probability distributions naturally appear as the solutions to the original equation of motion that has been derived from the first principles rather than taken a priori and hence has a clear physical meaning. In other words, the model is physically substantiated, and as such, unlike the purely mathematical approaches, it provides a physical explanation to the observed properties of real market data [33, 45].
3. These facts, together with the model's ability to forecast the direction of future price changes, suggest that one may expect a complete pricing model based on the current prototype to possess the desired permanent forecast strength suitable for practical purposes.

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