

A Financial Instruments Pricing Model as the physical two players problem^{*}

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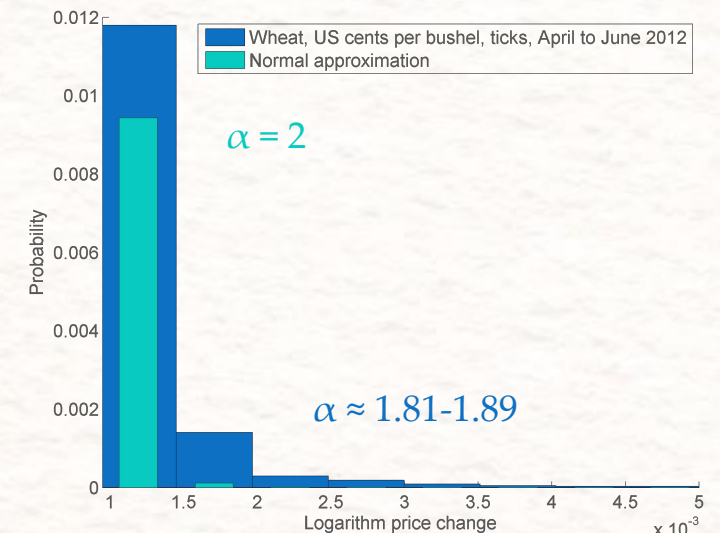
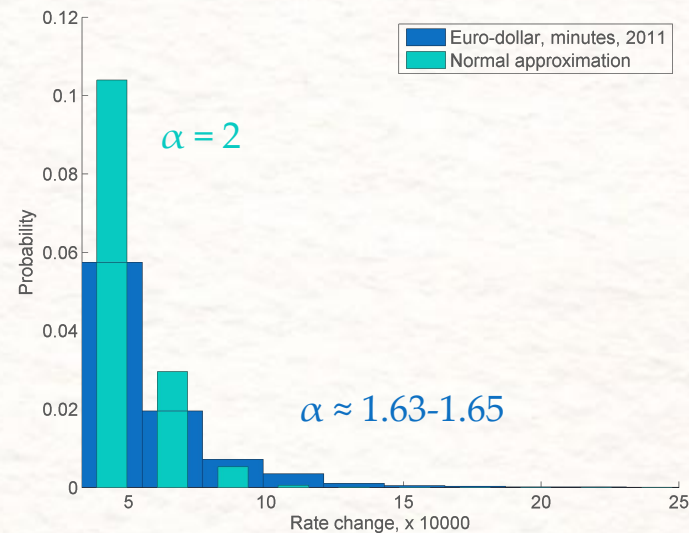
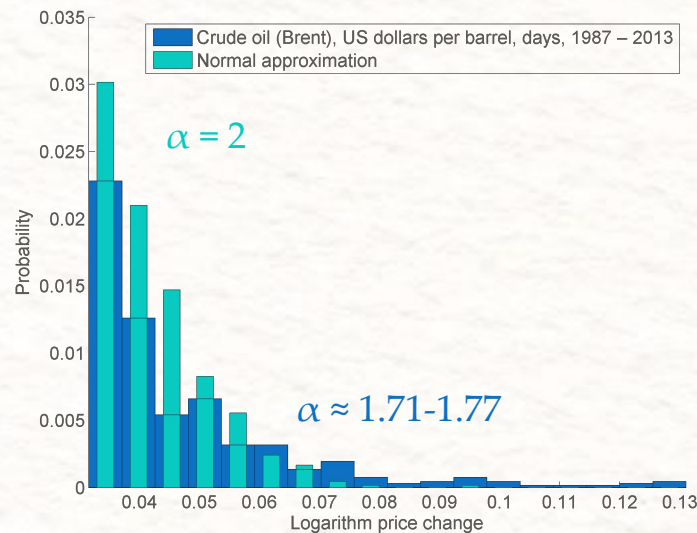
Fundamentals of the classical finance theory

- Louis Bachelier (1900), The theory of speculation
- Harry Markowitz (1952), The modern portfolio theory (MPT)
- William Sharpe (1964), The capital asset pricing model (CAPM)
- Fischer Black & Myron Scholes (1973), The option pricing model (OPM)

The common suppositions: price changes are *statistically independent* (the so-called “market memory” is absent) and distributed according to the *normal (Gaussian) law*

Benoit Mandelbrot's studies

In the 1960s Benoit Mandelbrot opined that price changes are *dependent* and distributed according to the *power laws*¹⁻⁵: $\Pr(|\Delta x| > x) \sim x^{-\alpha}, \quad x \rightarrow +\infty$



Data source: investment holding company "Finam", Russia (www.finam.ru)

Analysis of α is based on the algorithms by S. Kogon and D. Williams, *Characteristic-Function-Based Estimation of Stable Distribution Parameters*, in: R. Adler et al. (eds.), *A Practical Guide to Heavy Tails: Statistical Techniques and Applications*, Boston, Birkhauser, 1998, pp. 311–335 and I. Koutrouvelis, *Regression-type Estimation of the Parameters of Stable Laws*, J. Amer. Statist. Assoc., 69 (1980) 108–113

¹ B. Mandelbrot, *The Variation of Certain Speculative Prices*, J. Bus., 36 (1963) 394–419

² P. Cootner (ed.), *The Random Character of Stock Market Prices*, Cambridge, MA, MIT Press, 1964

³ E. Fama, *The Behavior of Stock-Market Prices*, J. Bus., 38 (1965) 34–105

⁴ B. Mandelbrot, *Fractals and Scaling in Finance*, N.Y., Springer, 1997

⁵ B. Mandelbrot, *The (Mis)Behavior of Markets*, N.Y., Basic Books, 2004

State of the art (econophysics)

There are various models imitating some or other statistical properties of the real market pricing process:

- Truncated Levy flight¹
- Modifications of the Schrodinger equation (“quantum” models)²
- Percolation models³
- Autoregressive heteroscedasticity (GARCH-based models) for volatility⁴
- Neural-network-based models⁵
- Ising-like (interacting agents) models⁶
- and others

¹ M. Morozova, *Options: Risk Reducing or Creating*, in: D. Sornette, S. Iylyev and H. Woodard (eds.), *Market Risk and Financial Markets Modeling*, Berlin, Springer, 2012, pp. 171–189

² O. Choustova, *Quantum Model for the Price Dynamics: The Problem of Smoothness of Trajectories*, *J. Math. Anal. Appl.*, 346 (2008) 296–304

³ H. Tanaka, *A Percolation Model of Stock Price Fluctuations*, *Mathematical Economics*, 1264 (2002) 203–218

⁴ T. Bollerslev, *Generalized Autoregressive Conditional Heteroskedasticity*, *J. Econometrics*, 31 (1986) 307–327

⁵ J.-Z. Wang, J.-J. Wang, Z.-G. Zhang and S.-P. Guo, *Forecasting Stock Indices with Back Propagation Neural Network*, *Expert Systems with Applications*, 38 (2011) 14346–14355

⁶ J. Voit, *The Statistical Mechanics of Financial Markets*, Berlin, Springer, 2005

State of the art (econophysics)

The imitation of the real pricing process is performed via taking an a priori given deterministic model and supplying it with a stochastic term(s)

Upon this, no physical justification on the choice of the model is provided

In doing so, the chance to guess the genuine model which would adequately describe the mechanism of real market pricing and hence possess permanent forecast strength is practically nil

Therefore, so far there is *no* physically justified statistical model which would provide an adequate description of the financial assets pricing dynamics¹

¹ R. Mantegna and H. Stanley, *Introduction to Econophysics: Correlations and Complexity in Finance*, Cambridge, Cambridge University Press, 2000

Our approach

- Market pricing is considered as an open stochastic mechanical system
- The entire set of market agents is deemed as two macroscopic¹ players (the “bull” and the “bear”), each “tugging” the price to the corresponding direction
- Market information that affects the price is likened to the energy introduced into the system

To find the law of dependence of the next price change Δx_{next} on the previous change Δx_{prev} , we formulate the original first principles:

- The more energy is required, the less probable the corresponding price change
- Price formation is being carried out simultaneously at diverse time scales

¹ H. Haken, *Information and Self-Organization: A Macroscopic Approach to Complex Systems*, Berlin, Springer, 2006

Our approach

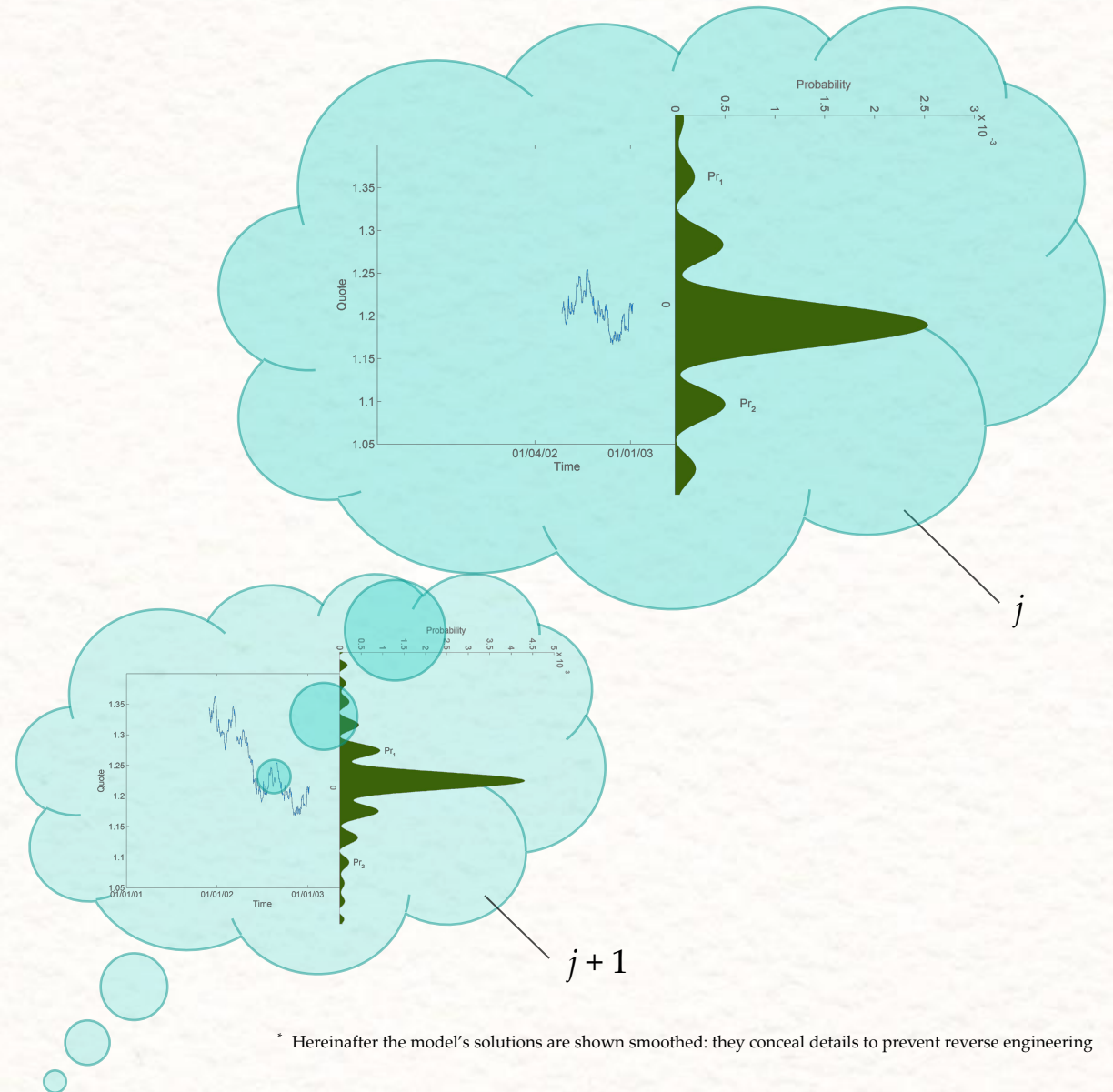
Our model* follows from the first principles rather than is taken a priori. It is an original probability equation

$$\Pr(j\Delta x) = \Pr(j\Delta x_{\text{next}} \mid j\Delta x_{\text{prev}})$$

written for each of the time scales j involved (analogous to minutes, hours, days, etc.)

Gathering all over the involved scales to form the price is what we call the *scales complexification*:

$$\Pr(j\Delta x) = \xi(\Pr(j\Delta x), \dots, \Pr(j^{+N}\Delta x))$$



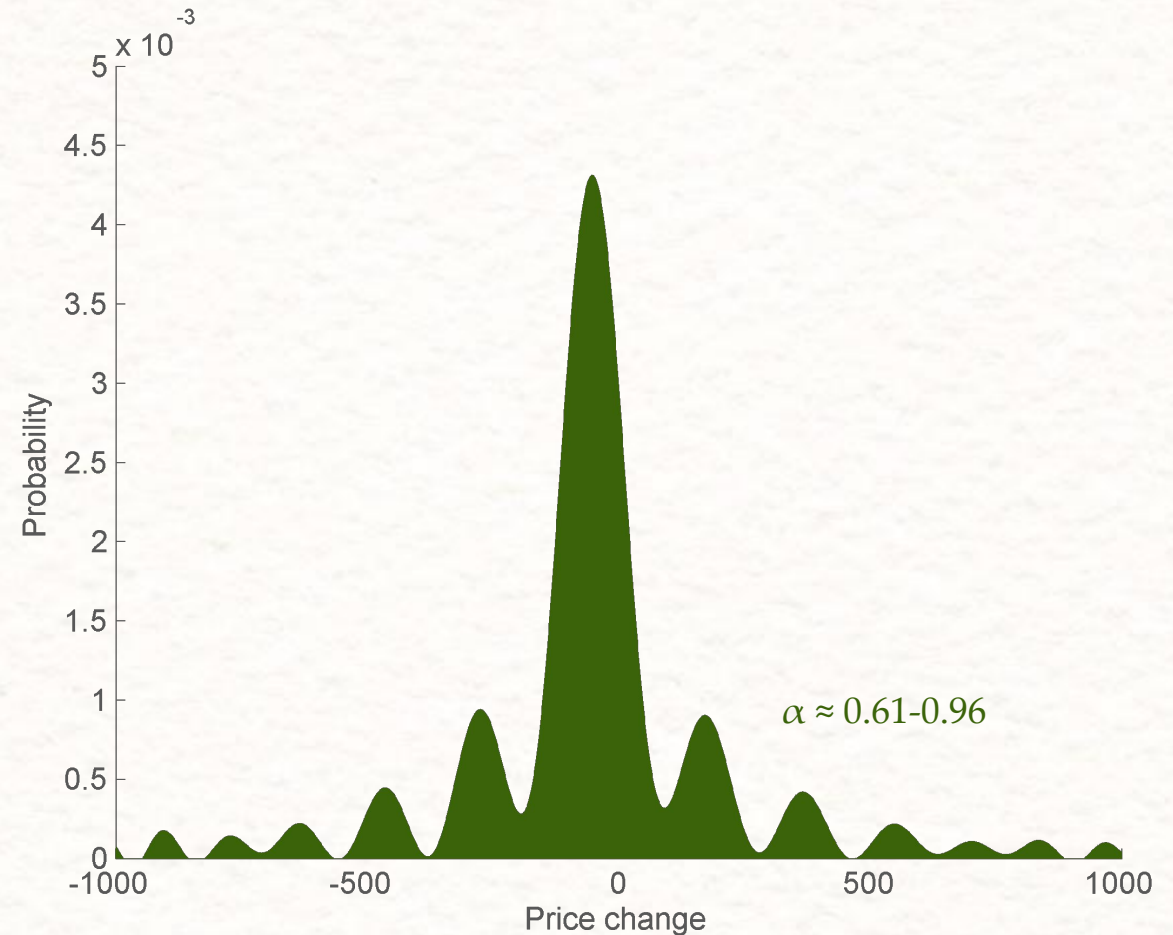
* Hereinafter the model's solutions are shown smoothed: they conceal details to prevent reverse engineering

Our results

At each time scale j the solution to the equation

$$\Pr(j\Delta x) = \Pr(j\Delta x_{\text{next}} \mid j\Delta x_{\text{prev}})$$

is a wave-like asymmetric distribution with *fat tails*¹



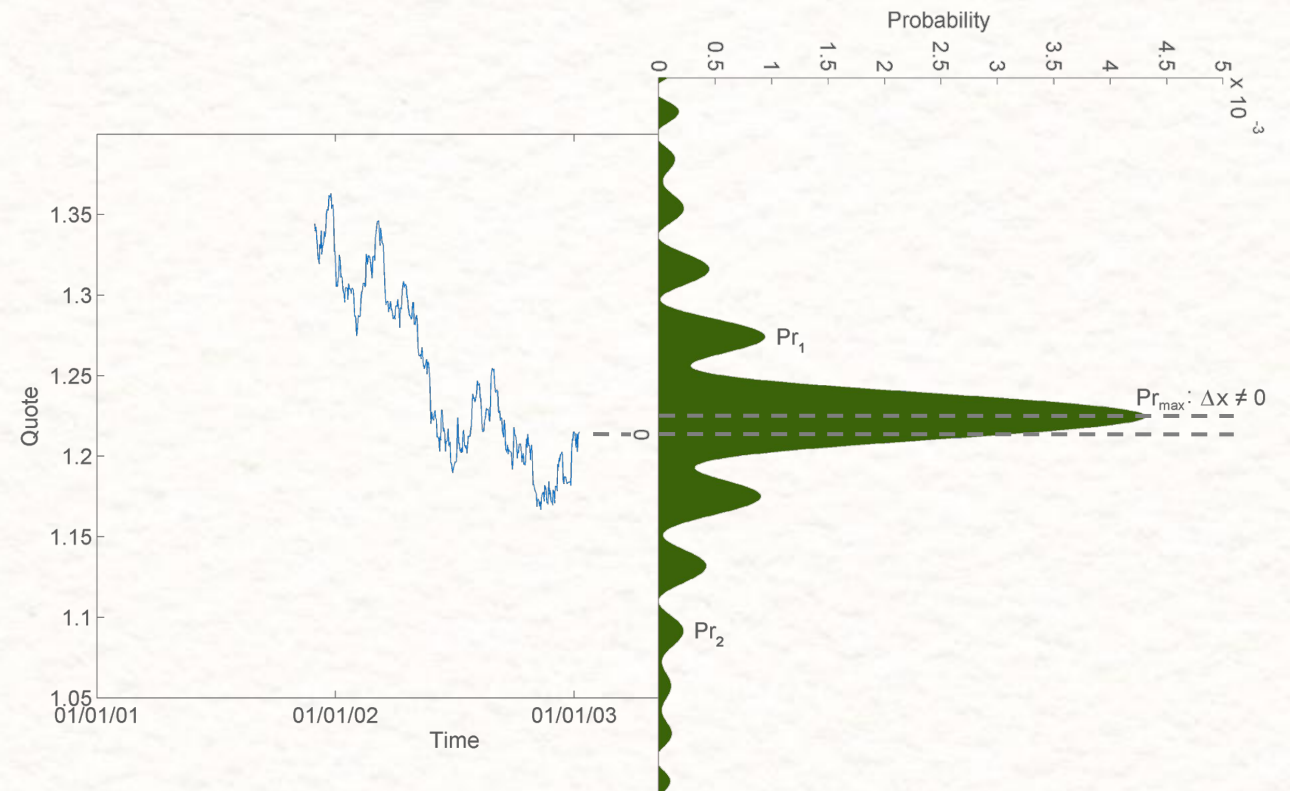
¹ B. Mandelbrot, *Fractals and Scaling in Finance*, N.Y., Springer, 1997

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Our results

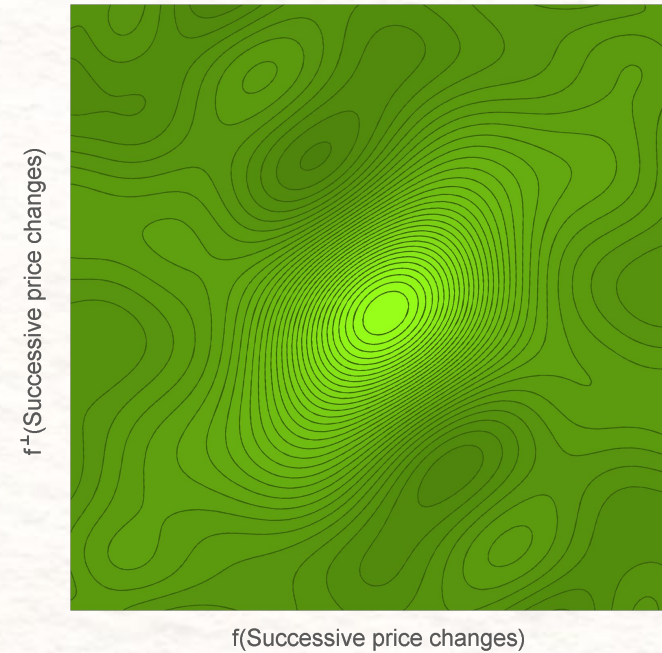
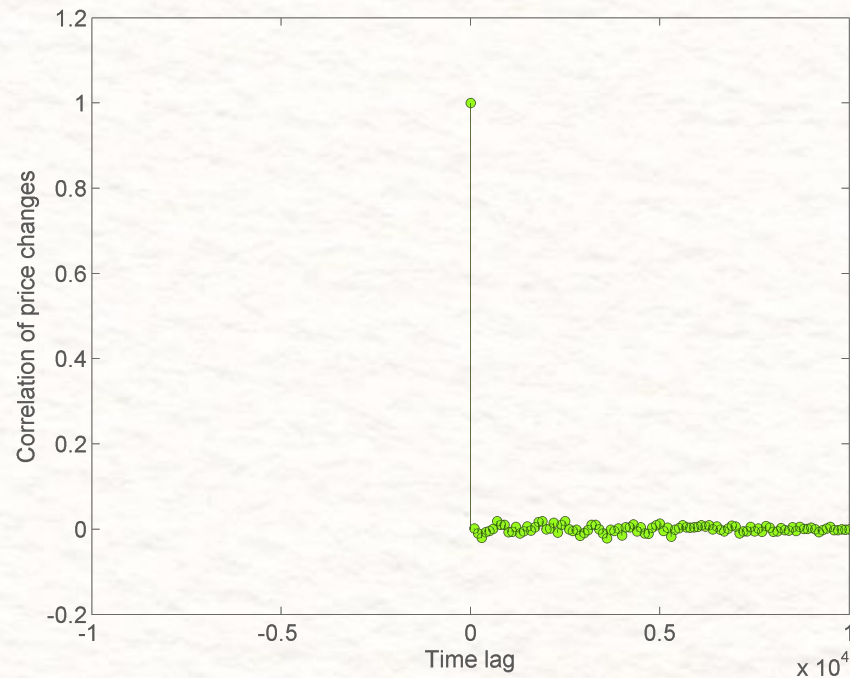
The asymmetry of the probability distribution allows forecasting not only the future volatility, but also the direction of the next price change ${}^j\Delta x_{\text{next}}$ subject to the known previous one ${}^j\Delta x_{\text{prev}}$:

$$\Pr({}^j\Delta x) = \Pr({}^j\Delta x_{\text{next}} \mid {}^j\Delta x_{\text{prev}})$$



Our results

The asymptotic symmetry of the distribution clarifies the empirical zero correlation of price changes¹⁻³ and asserts the presence of nonlinear *market memory*⁴



¹ E. Fama and M. Blume, *Filter Rules and Stock-Market Trading*, J. Bus., 39 (1966) 226–241

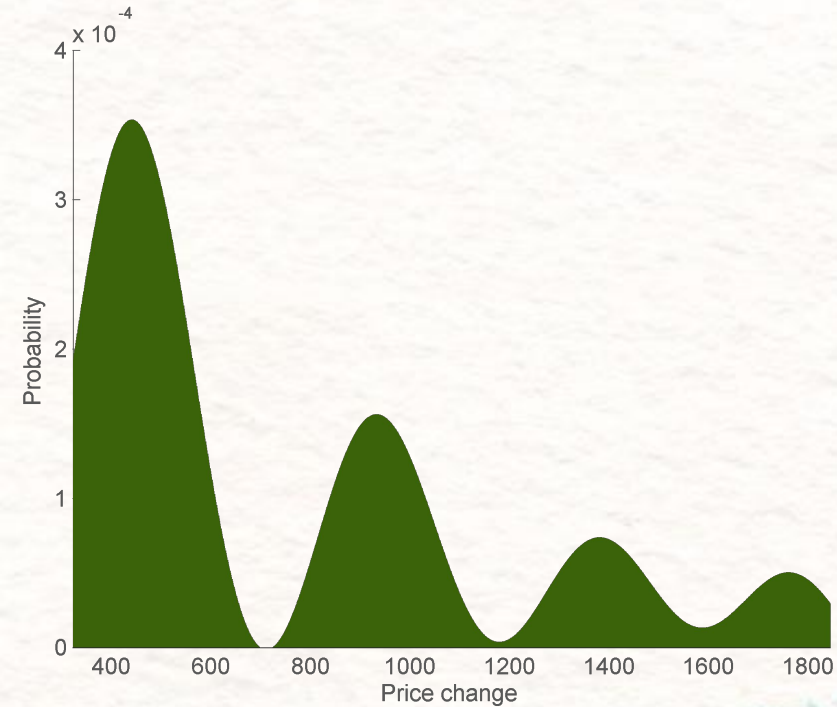
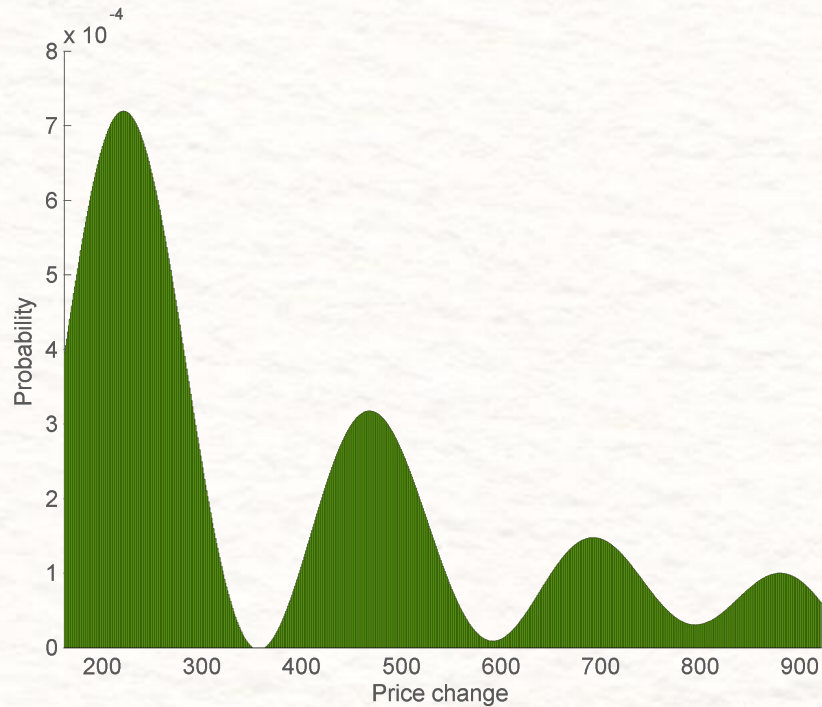
² R. Mantegna and H. Stanley, *Introduction to Econophysics: Correlations and Complexity in Finance*, Cambridge, Cambridge University Press, 2000

³ P. V. Vidov and M. Yu. Romanovskiy, *Nonclassical Random Walks and the Phenomenology of the Securities Yield Fluctuations on the Stock Market*, Uspekhi Phys. Nauk, 7 (2011) 774–778 (in Russian)

⁴ B. Mandelbrot, *Fractals and Scaling in Finance*, N.Y., Springer, 1997

Our results

The model's solution has the property of *scaling*¹ – the structure of the distribution is kept unchanged when passing from one scale to another



¹ B. Mandelbrot, *Fractals and Scaling in Finance*, N.Y., Springer, 1997

Our results

To check for the forecast strength, we have tested our model on real tick data over three years and artificial (random walk) data

The statistically significant skewness $\text{Fr}(\text{sgn } \Delta x_{\text{next}} \mid \Delta x_{\text{prev}}) - 50\%$ in the relative frequency of the right forecast of the direction of the future price change evinces that the model *detects a dependence* in the data

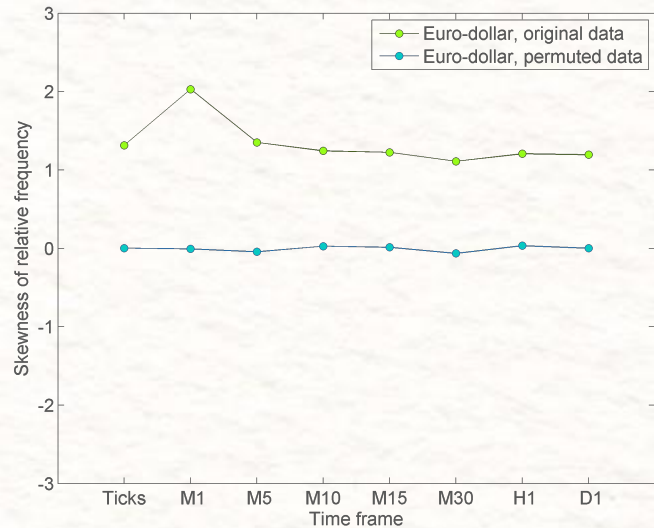
Asset	$\text{Fr}(\text{sgn } \Delta x_{\text{next}} \mid \Delta x_{\text{prev}}) - 50\%$	
	original data	permuted data
EUR/USD ¹	1.32	0.00
Oil ²	2.26	-0.00
Gold ²	2.84	-0.01
Wheat ²	4.26	-0.05
Google ²	-1.10	-0.04
Apple ²	-1.48	0.04
Random walk	0.02	-0.04

Data source: ¹ Forex bank "Dukascopy", Switzerland (www.dukascopy.com)

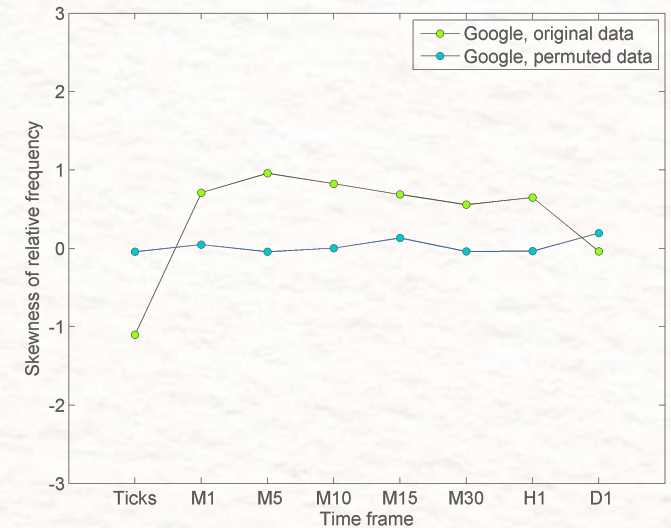
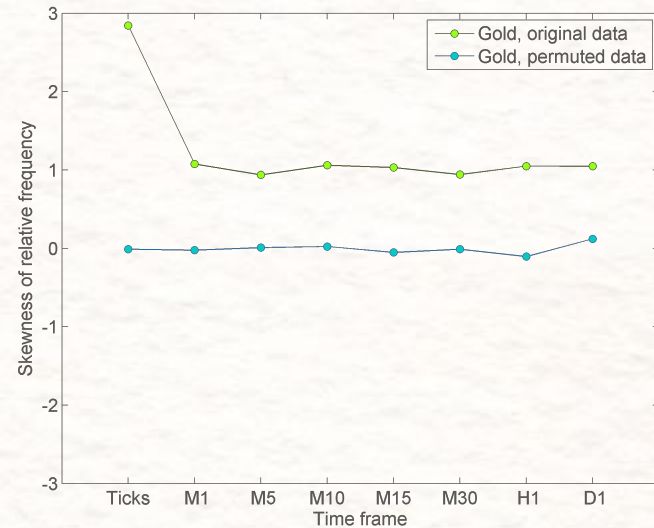
² investment holding company "Finam", Russia (www.finam.ru)

Our results

Similar results take place when processing minutely, hourly and daily quotes



Data source: investment holding company "Finam", Russia (www.finam.ru)



Conclusion

Analysis of the properties of the developed model allows to conclude:

- The model's solution – the *fat-tailed* probability distributions that possess *scaling* and detect nonlinear *market memory* – is consistent with the well-known empirical facts about the properties of real market data¹⁻³

What differs our model from the others is that:

- It has been derived from the first principles (“with the tip of the pen”) rather than taken a priori
- It provides a physical explanation to the observed properties of real market data
- It is able to make forecast about the direction of future price changes

¹ B. Mandelbrot, *Fractals and Scaling in Finance*, N.Y., Springer, 1997

² R. Mantegna and H. Stanley, *Introduction to Econophysics: Correlations and Complexity in Finance*, Cambridge, Cambridge University Press, 2000

³ J. Voit, *The Statistical Mechanics of Financial Markets*, Berlin, Springer, 2005

Appendix: Possible applications

A complete market pricing model based on the current prototype may be the basement for the following applications:

- Proprietary trading
- Estimation of market risks (e.g., value at risk, expected tail loss, etc.) and providing consulting services to financial institutions
- Creation of exchange-traded funds for the attraction of investments and subsequent portfolio management
- ?

Appendix: What remains to do

The following tasks have yet to be completed to make the end-user product:

R&D stage:

- To supply the model with control parameters¹ which would make it possible to simulate the behaviour of financial assets of different types
- To determine the length of the market memory and the values of the control parameters for each particular asset forming the portfolio

Programming stage:

- To develop a software which would implement a set of auxiliary technical tools (real time market data flow, graphical user interface, etc.)

¹ H. Haken, *Information and Self-Organization: A Macroscopic Approach to Complex Systems*, Berlin, Springer, 2006

Thank you