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Modelling of combustion and diverse blow-up regimes in a spherical shell

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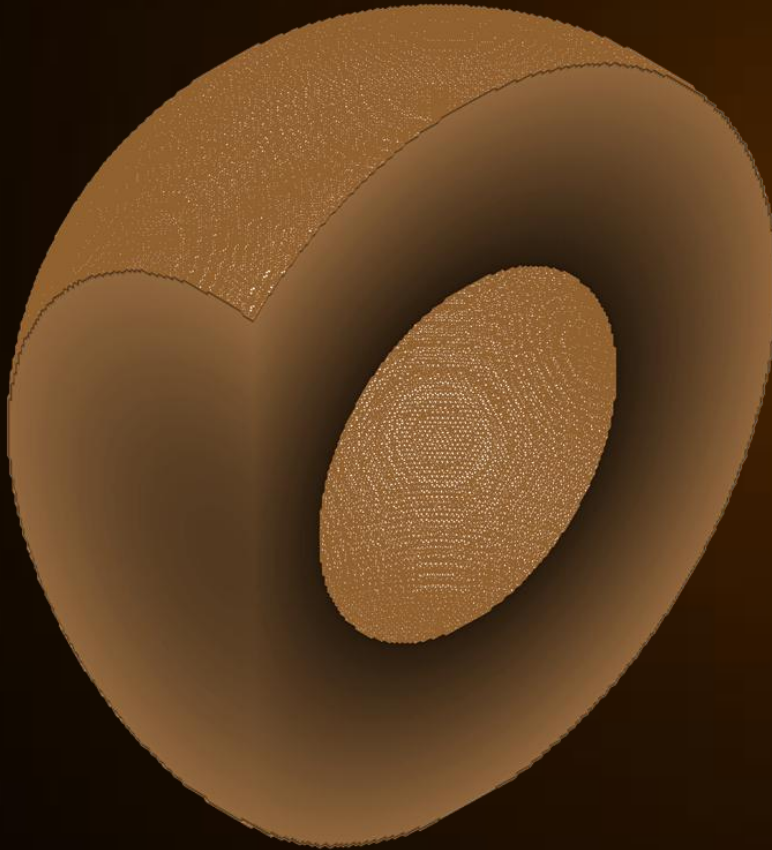
Introduction

Evolution towards blow-up takes place in many important applications, e.g.

- thermonuclear fusion
- inelastic deformation and fracture of solids
- autocatalytic processes in chemical reactors
- weather dynamics (energy transfer in tornado and lightning)
- etc.

Problem formulation

An important physical process revealing different blow-up modes is the phenomenon of combustion described by the nonlinear diffusion equation



$$\begin{cases} \frac{\partial T}{\partial t} = (A_r + A_\lambda + A_\varphi)T + f, \\ T|_{t=0} = T^0, \end{cases}$$

External forcing is responsible for energy accumulation

$$A_r T \equiv \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 D \frac{\partial T}{\partial r} \right),$$

$$A_\lambda T \equiv \frac{1}{r^2 \cos \varphi} \frac{\partial}{\partial \lambda} \left(\frac{D}{\cos \varphi} \frac{\partial T}{\partial \lambda} \right),$$

$$A_\varphi T \equiv \frac{1}{r^2 \cos \varphi} \frac{\partial}{\partial \varphi} \left(D \cos \varphi \frac{\partial T}{\partial \varphi} \right),$$

Diffusion coefficient is responsible for energy dissipation

$$D \equiv \mu T^\alpha, \quad \alpha \geq 0,$$

$$f \sim T^\beta, \quad \beta > 0.$$

Solution

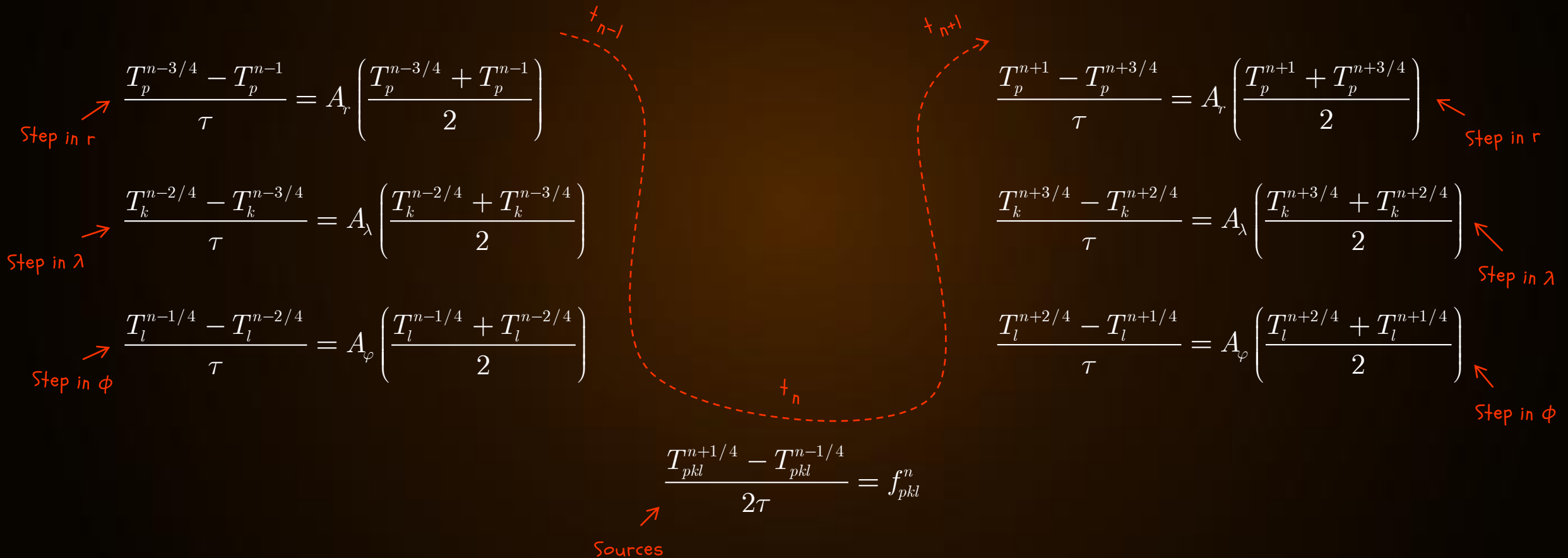
Smart grid stencil



Because of the term $\cos^{-1}\varphi$ we shift the spherical grid a half step in λ and φ to avoid the pole singularities

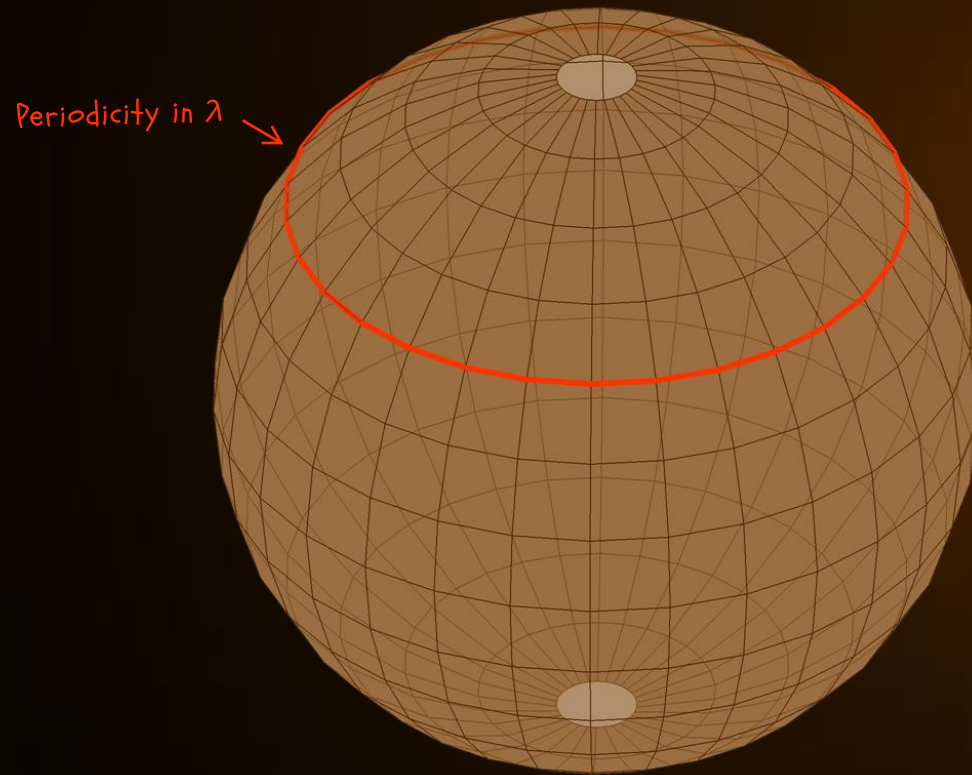
Solution

Operator splitting



Solution

Grid swapping

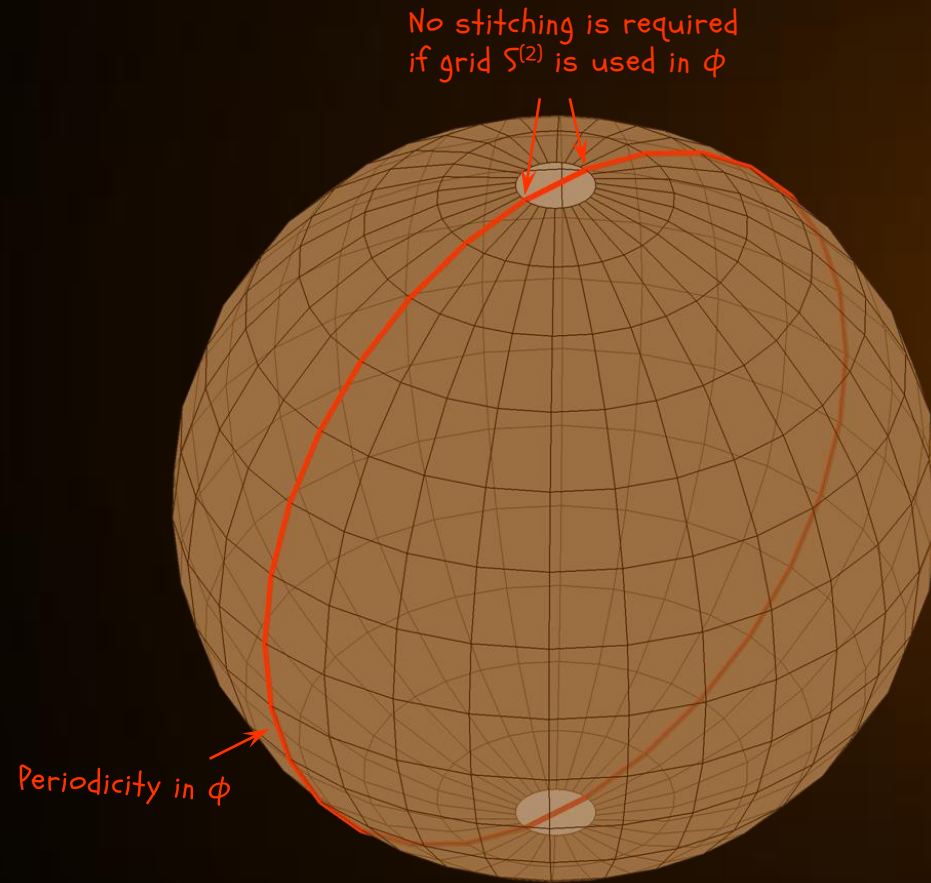


Due to the grid swapping we use periodic boundary conditions in λ and φ , and thereby avoid the challenge of stitching the solution near the poles

$$S^{(1)} = \left\{ (\lambda_k, \varphi_l) : \lambda_k \in \left[\frac{\Delta\lambda}{2}, 2\pi + \frac{\Delta\lambda}{2} \right), \varphi_l \in \left[-\frac{\pi}{2} + \frac{\Delta\varphi}{2}, \frac{\pi}{2} - \frac{\Delta\varphi}{2} \right] \right\}$$

Solution

Grid swapping



Due to the grid swapping we use periodic boundary conditions in λ and φ , and thereby avoid the challenge of stitching the solution near the poles

$$S^{(2)} = \left\{ (\lambda_k, \varphi_l) : \lambda_k \in \left[\frac{\Delta\lambda}{2}, \pi - \frac{\Delta\lambda}{2} \right], \varphi_l \in \left[-\frac{\pi}{2} + \frac{\Delta\varphi}{2}, \frac{3\pi}{2} + \frac{\Delta\varphi}{2} \right] \right\}$$

Solution

Finite difference schemes

$$\frac{\partial}{\partial r} \left(r^2 D \frac{\partial T}{\partial r} \right) \approx \frac{1}{\Delta r} \left((r^2 D)_{p+\frac{1}{2}} \frac{T_{p+1} - T_p}{\Delta r} - (r^2 D)_{p-\frac{1}{2}} \frac{T_p - T_{p-1}}{\Delta r} \right) \quad \text{Second order in } r$$

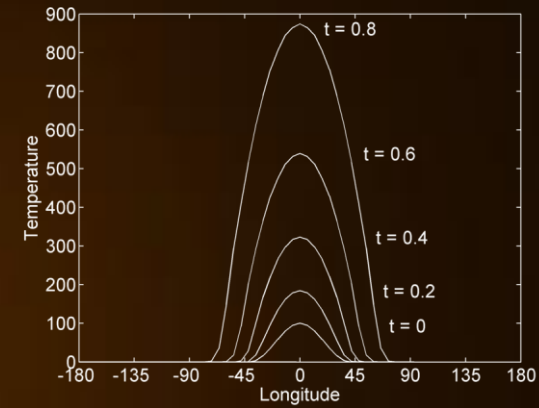
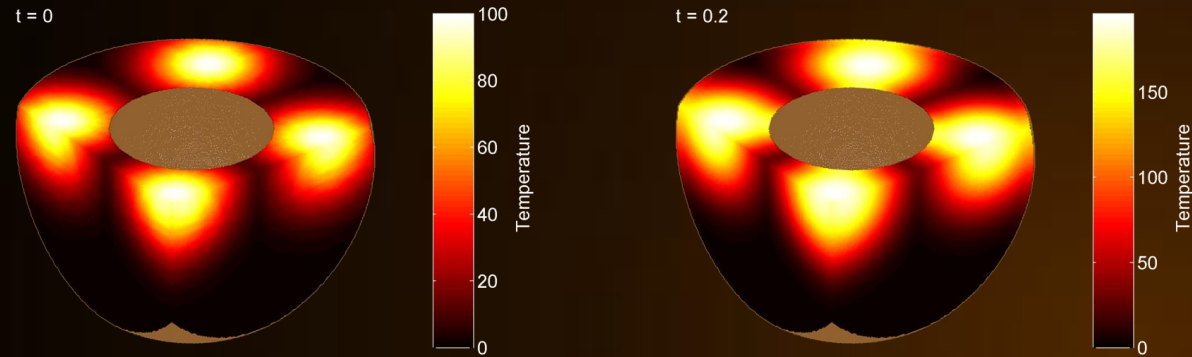
$$\frac{\partial}{\partial \lambda} \left(D \frac{\partial T}{\partial \lambda} \right) \approx \frac{1}{\Delta \lambda} \left(D_{k+\frac{1}{2}} \frac{T_{k+1} - T_k}{\Delta \lambda} - D_{k-\frac{1}{2}} \frac{T_k - T_{k-1}}{\Delta \lambda} \right) \quad \begin{array}{l} \text{Second order in } \lambda \\ \text{(arbitrary even} \\ \text{order is possible)} \end{array}$$

$$\frac{\partial}{\partial \varphi} \left(D |\cos \varphi| \frac{\partial T}{\partial \varphi} \right) \approx \frac{1}{\Delta \varphi} \left((D |\cos \varphi|)_{l+\frac{1}{2}} \frac{T_{l+1} - T_l}{\Delta \varphi} - (D |\cos \varphi|)_{l-\frac{1}{2}} \frac{T_l - T_{l-1}}{\Delta \varphi} \right) \quad \begin{array}{l} \text{Second order in } \phi \\ \text{(arbitrary even} \\ \text{order is possible)} \end{array}$$

The resulting finite difference schemes are dissipative
and possess the property of balance (mass conservation)

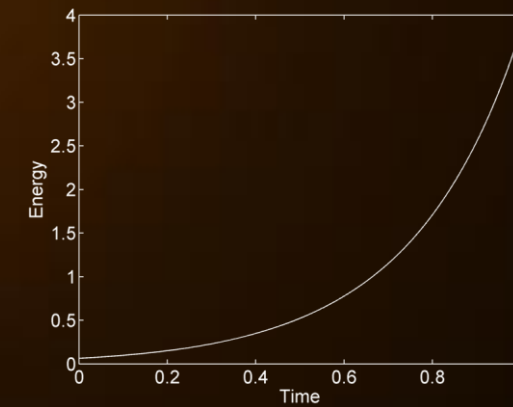
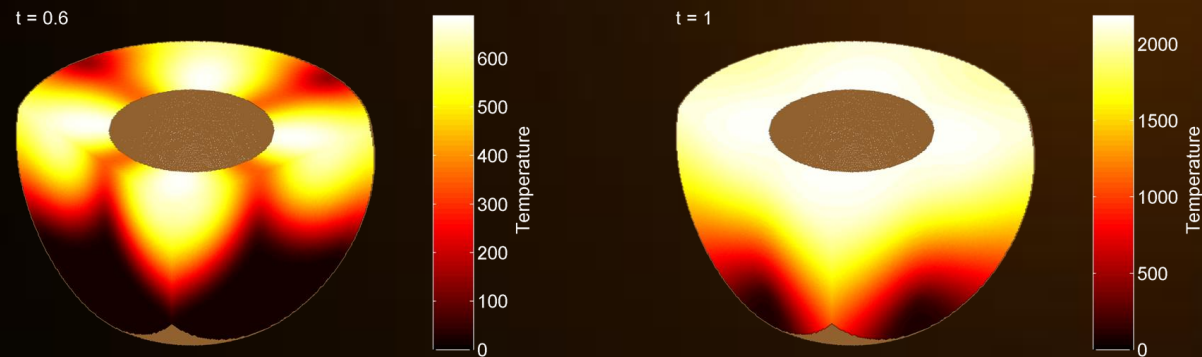
Numerical results

Combustion (HS-mode)



$$D \sim T^\alpha,$$
$$f \sim T^\beta,$$
$$\beta < \alpha + 1$$

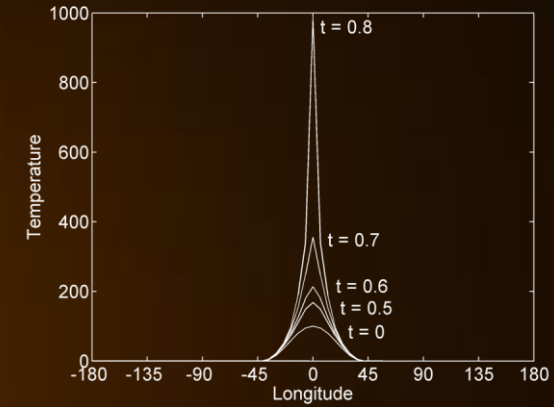
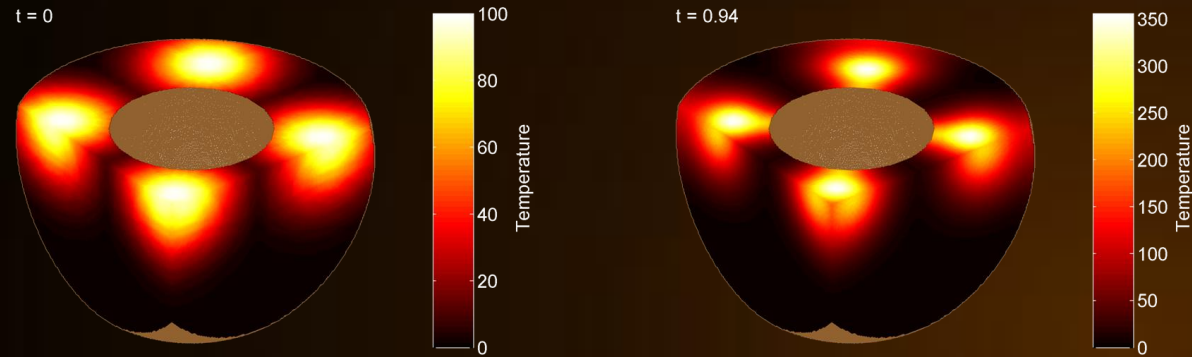
Area of combustion is expanding with time



Blow-up energy growth

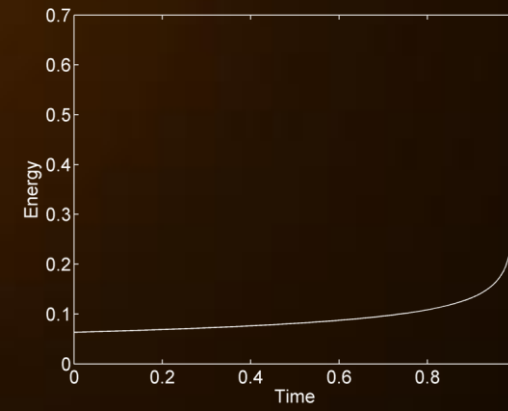
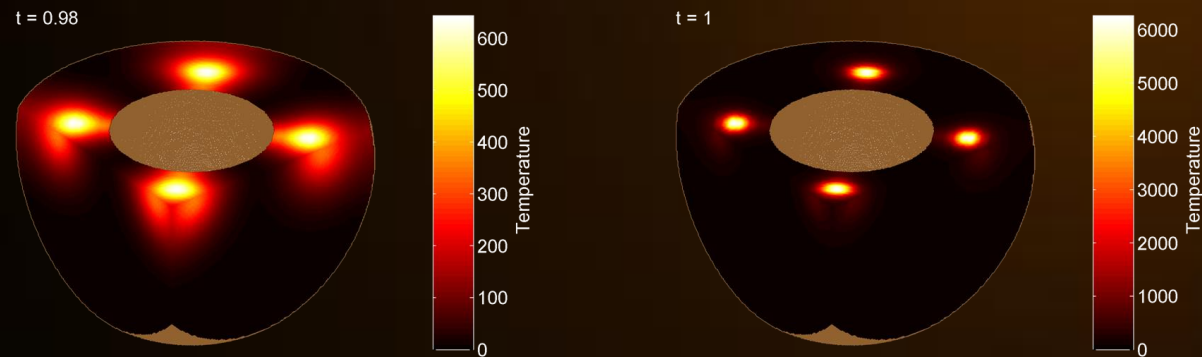
Numerical results

Combustion (LS-mode)



$$D \sim T^\alpha,$$
$$f \sim T^\beta,$$
$$\beta > \alpha + 1$$

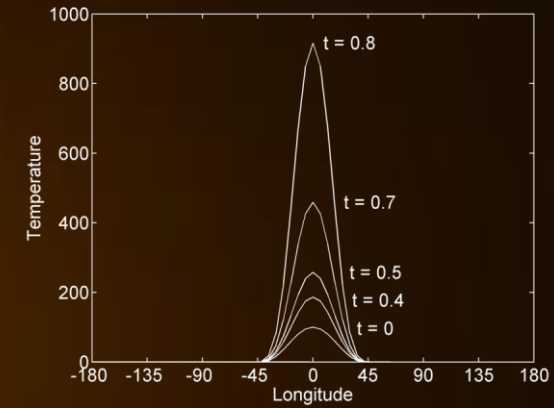
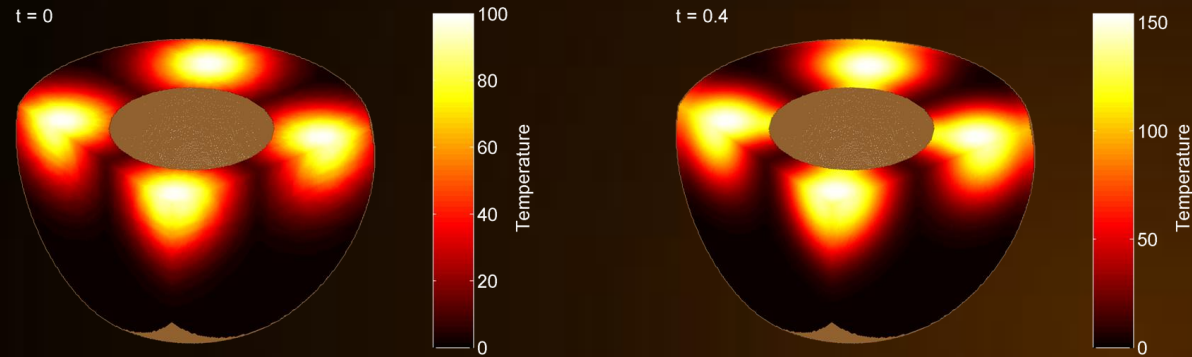
Area of combustion
is reducing with time



Blow-up
energy growth

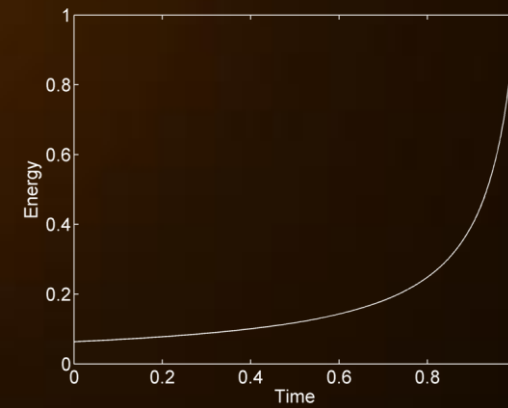
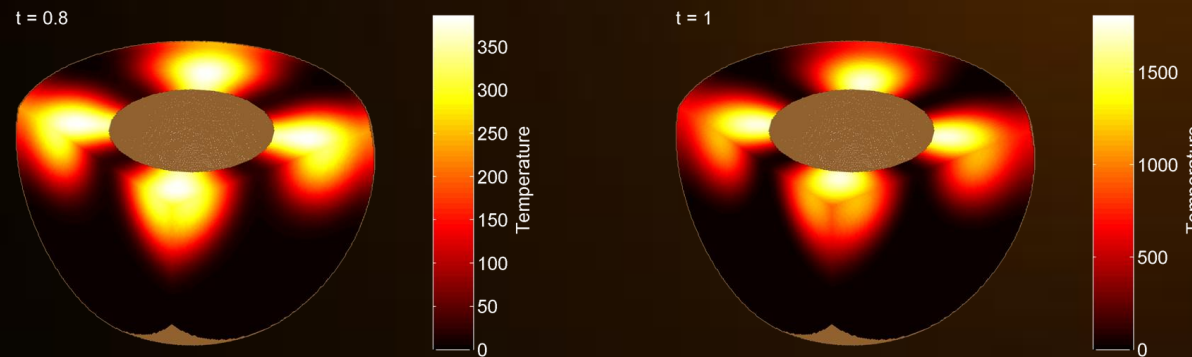
Numerical results

Combustion (S-mode)



$$D \sim T^\alpha,$$
$$f \sim T^\beta,$$
$$\beta = \alpha + 1$$

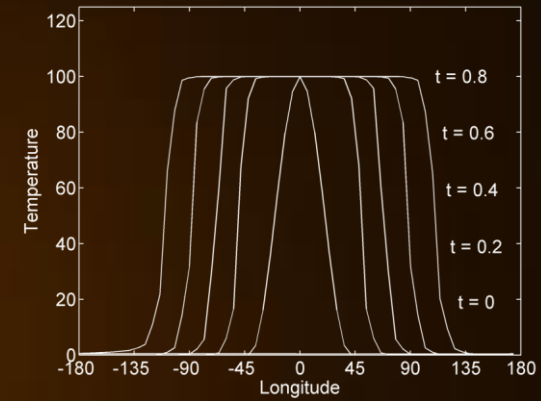
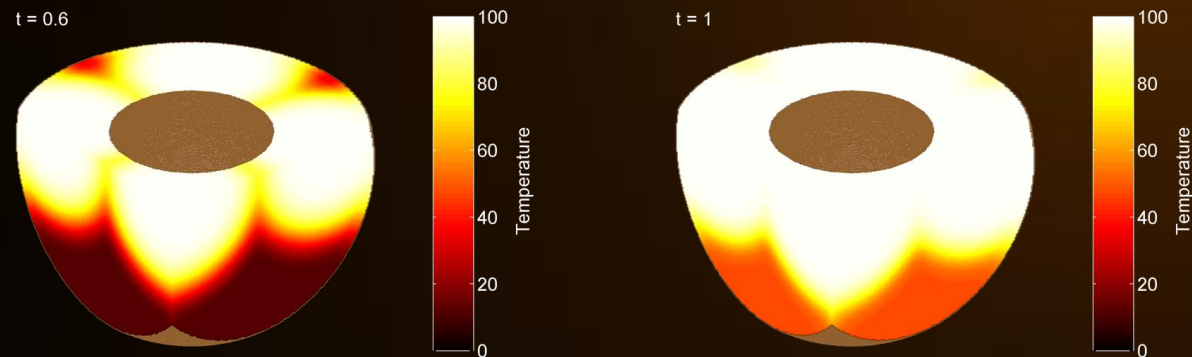
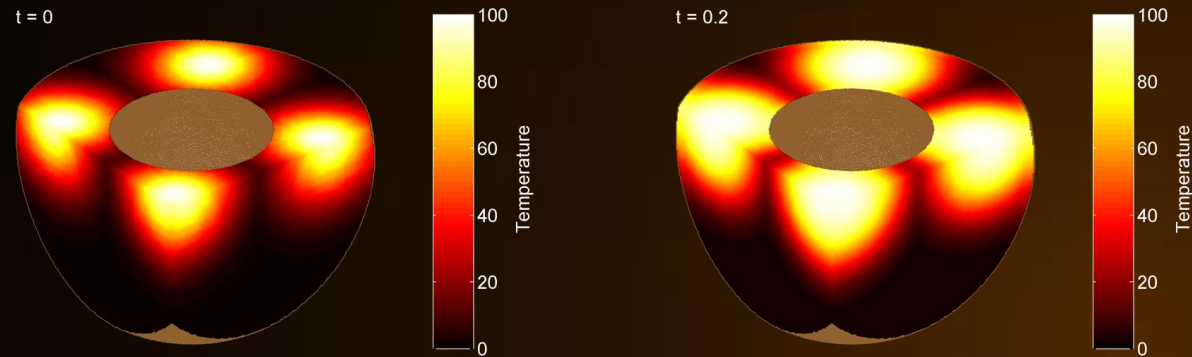
Area of combustion is kept nearly constant



Blow-up energy growth

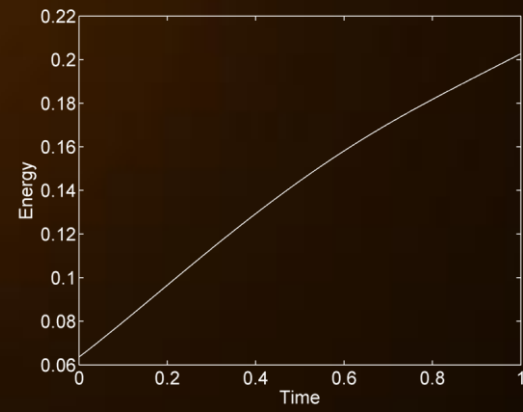
Numerical results

Temperature waves



$$D = \text{const},$$
$$f \sim T - T^3$$

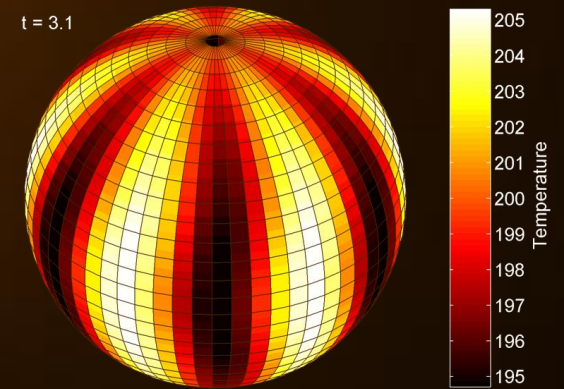
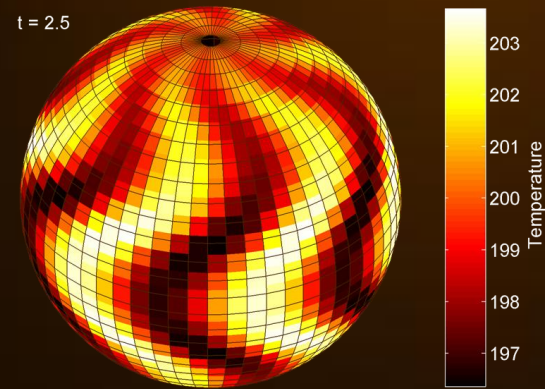
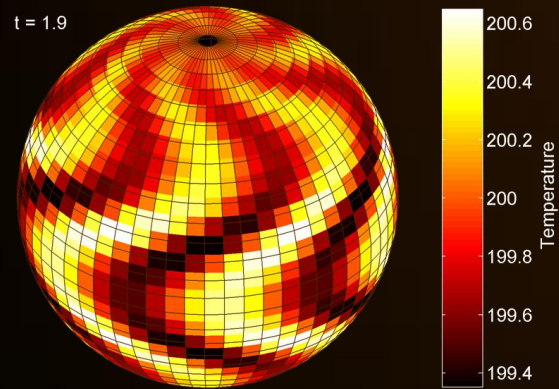
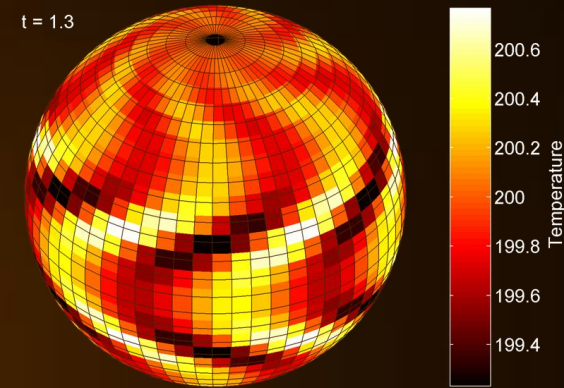
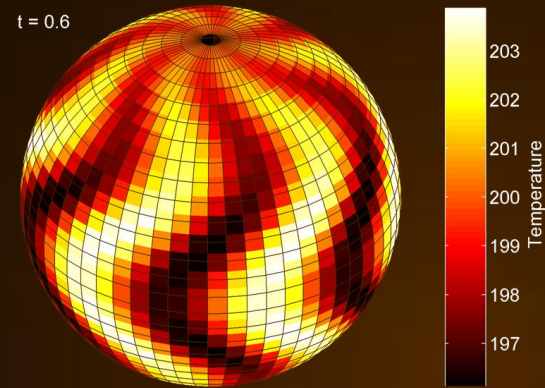
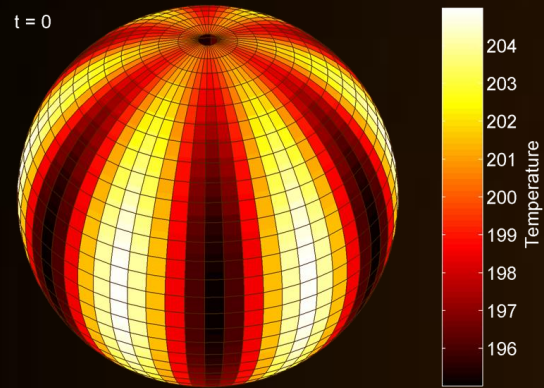
Transfer of energy with
constant wave front



Quiet
energy growth

Numerical results

Spherical travelling waves



$$D \sim T^\alpha,$$
$$f = f(\lambda, \varphi, t)$$

↑
Transfer of energy via
oscillating sources

Conclusion

Due to the techniques of operator splitting and grid swapping we have avoided difficulties related to the spherical geometry and hence *developed fast and easy-to-implement numerical method*

The developed method allows highly accurate simulation of various nonlinear diffusion phenomena in complex media and *permits parallel implementation on high-performance systems*

Our simulator can be used for studying energy transfer towards blow-ups in many practically important applications, e.g. thermonuclear fusion, inelastic deformation and fracture of solids, autocatalytic processes in chemical reactors, etc., which is *relevant for energy science, materials science*, among others.

Thank you!

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